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An application of the Ryll-Nardzewski–Woyczyński theorem to a uniform weak law for tail series of weighted sums of random elements in Banach spaces

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Abstract

For a sequence of Banach space valued random elements $\{V_n, n\geqslant 1\}$ (which are not necessarily independent) with the series $\sum_{n=1}^{\infty} V_n$ converging unconditionally in probability and an infinite array $a=\{a_{ni},\ i\geqslant n,\ n\geqslant 1\}$ of constants, conditions are given under which (i) for all $n\geqslant 1$, the sequence of weighted sums $\sum_{i=n}^{m} a_{ni}V_i$ converges in probability to a random element $T_n(a)$ as $m\to\infty$, and (ii) $T_n(a)\xrightarrow{\mathrm{P}}0$ uniformly in a as $n\to\infty$ where a is in a suitably restricted class of infinite arrays. The key tool used in the proof is a theorem of Ryll-Nardzewski and Woyczyński (1975, Proc. Amer. Math. Soc. 53, 96–98). © 2000 Elsevier Science B.V. All rights reserved

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1. Introduction

In this paper, a uniform weak law of large numbers (WLLN) is established for tail series of weighted sums of Banach space valued random elements. Throughout, $\{V_n, n\geqslant 1\}$ is a sequence of random elements defined on a probability space $(\Omega,\mathscr{F},P)$ and assuming values in a real separable Banach space $\mathscr{X}$ with norm $\|\cdot\|$. In general, it is not assumed that the $\{V_n, n\geqslant 1\}$ are independent.

Let $\Theta=\{\theta=(\theta_1,\theta_2,\ldots),\theta_n=\pm 1, n\geqslant 1\}$. The series $\sum_{n=1}^{\infty} V_n$ is assumed to *converge unconditionally in probability*; that is, for every $\theta\in\Theta$, the sequence of partial sums $S_n^{\theta}\equiv\sum_{i=1}^{n}\theta_i V_i$ converges in probability to

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